

Research Article

Exergy Analysis of a Reverse Stirling Cycle Cryogenerator for Generating Liquid Nitrogen

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Abstract

An Exergy analysis is performed to compute the energy saving of the entire Reverse Stirling Cryogenerator system for liquefaction of nitrogen gas. The methodology of exergy analysis is based on entropy changes of different components and is an effective tool for finding optimal operating parameters. Exergy balance indicates the concept of exergy efficiency which takes into attention the quality of energy. This analysis helps pointing out the piece of equipments requiring improvement in efficiency. The paper carries out systematic thermodynamics analysis to compute specific work requirement of refrigerant gas, overall thermodynamic efficiency, specific Cooling capacity, energy required per kg of liquid nitrogen product and Coefficient of Performance of the refrigeration cycle using separately hydrogen and helium as refrigerant gas. The results are found to be realistic and encouraging.

Keywords: Exergy analysis, Reverse Stirling cycle, Cryogenerator, Coefficient of Performance, Liquid Nitrogen generation

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Introduction

The Cryogenerator based on Reverse Stirling Cycle have been successfully utilized for liquefaction of cryogenic gases for wide areas of applications in the large to small scale plant. The exergy method will readily account for the irreversible losses in the system that must be considered for realistic performance evaluation, especially for an energy conservation analysis. Exergy identifies the major sources of losses and areas for improving the performance of the system. Exergy analysis is based on second law of thermodynamics and more specifically an entropy analysis means the available work energy function. The exergy of a system at a given state can be defined as the maximum work that would be obtained in taking the system to the 'dead state (normally, the ambient) for heat engine. For refrigeration system, exergy is minimum work required to take the system to a given state from its dead state. The methodology of energy analysis can be useful for selecting optimal operating parameters and for comparing different process configurations. Energy balance leads to the concept of energetic efficiency which takes into consideration the quality of energy [1].

Chen-Hwa Chiu [2] in his work discussed the advantages and limitations, iterative and analytical comparative of applications of exergy analysis to cryogenic process and equipment's. Exergy method of analysis for variation of parameters is applied on different cycle configuration for pressure swing adsorption system [3]. The methodology of exergy analysis provides a rational criterion for determination of the optimal operating parameters for a specified configuration and for comparing different configurations [4]. Various exergy analytical approaches were also reported in the literature [5-15]. The techniques are also used for Liquefied Natural gas (LNG) system [16-21]. Investigator also analyzed cryocooler, Stirling Engine, Heat pumps, Carnot refrigerator [22-27].

The present work stresses on closed loop and high pressure refrigeration cycle with Hydrogen or Helium as working fluid. The cold generated in the refrigeration cycle is to be utilized for liquefaction of purified nitrogen which passes through a separate passage in heat exchanger i.e. condenser. A detail of thermodynamic analysis, exergetic losses and performance study of the cycle is carried out for better insight into the optimum design of such small scale nitrogen liquefier.

Process of Reverse Stirling Cycle Cryogenerator

Repeated compression and expansion of perfect gases like Hydrogen / Helium in Reverse Stirling Cycle generates cold for liquefaction of gases. The system, in principle, consists of a cylinder, a piston, a displacer and a regenerator. The piston compresses the gas while the displacer simply moves the gas from one chamber to another without changing the gas volume. The heat exchange during the constant volume is carried out in the regenerator.

Cryogenerator is the main section of the plant, which comprises compressor, refrigerator, regenerator, liquefiers all of them mounted on a common bases plate with a motor. The low temperature for gas liquefaction is obtained at the top space of the compressor adjacent to the exchanger of the liquefier due to periodic expansion of refrigerant gas within that space [28, 29]. The refrigeration Cycle follows the four steps as shown in **Figure 1**.

- Isothermal compression of working fluid where heat is released to the surroundings by cooling water. (Step :1-2)
- The expander is moved towards the piston, forcing the compressed gas through the regenerator where it is cooled. The energy removed is stored in the regenerator. (Step :2-3)
- The isothermal expansion involves both displacer and piston in which heat is absorbed at the low temperature. (Step :3-4)
- The displacer/expander moves away from the piston helping the gas to pass through the regenerator. The energy previously stored in the regenerator warms the refrigerant gas to bring it to its initial temperature before its compression. (Step :4-1)

Description of the Proposed Cryogenerator expander cycle for Nitrogen Liquefaction

The proposed high pressure liquefaction cycle for nitrogen gas is presented in (**Figure 2**). Gas such as Helium or Hydrogen is used as the refrigerant in the closed loop refrigerating cycle which produces the required cooling effect for liquefaction of nitrogen gas.

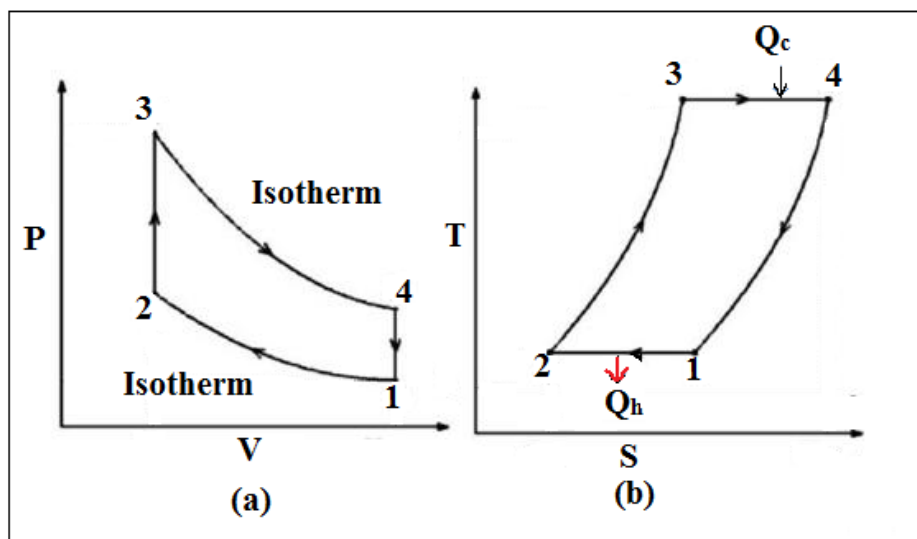


Figure 1 Stirling Refrigeration Cycle (a) P-V diagram (b) T-S diagram

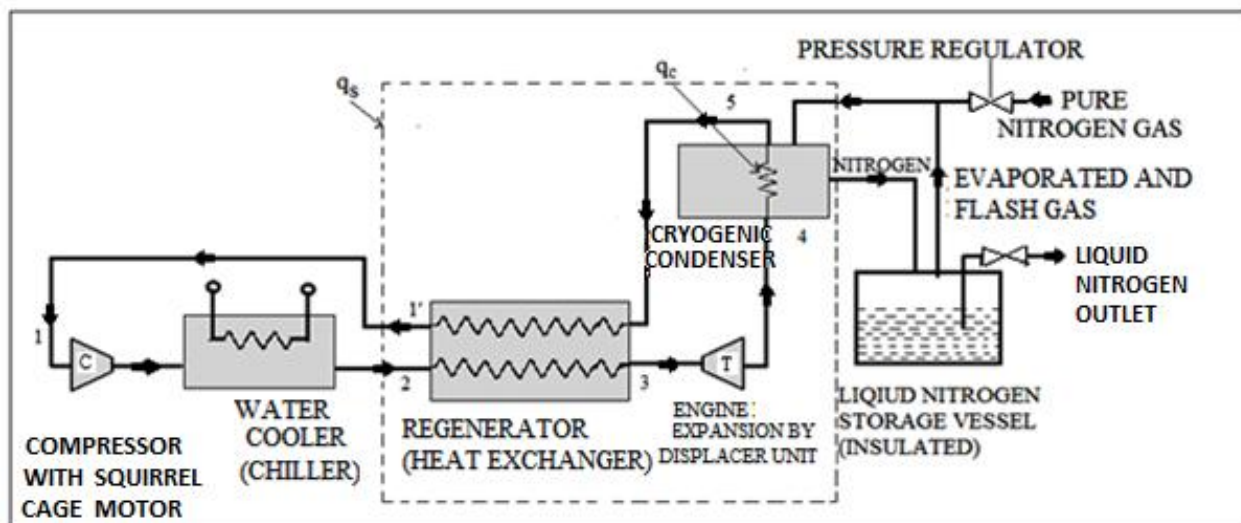


Figure 2 Stirling cycle based simple Expander cycle for nitrogen liquefaction

The gas is isothermally compressed from 0.1 MPa (1 atm.) and ambient temperature (T_1) to a higher pressure at about 2.22 MPa (22atm) in the compressor C and work performed by compressor is W_c . The gas is then admitted to a heat exchanger at temperature $T_2 = T_1$. The gas comes out from the heat exchanger at point 3 and its temperature drops down to T_3 . Past the heat exchanger the gas is admitted in a turbo expander T, where it expands to a low pressure while performing some work W_e . The pressure ratio of the expander is adjusted in such a way so that the refrigerant gas does not reach its dew point at the expander exit at the prevailing low pressure. The cold expanded gas goes to the condenser, low temperature condenser (LTC) where the refrigerant gas absorbs heat q_c from the pure nitrogen gas and which in turn gets liquefied. The temperature of refrigerant is raised from T_4 corresponding to point 4 to T_5 (point 5). The return stream of refrigerant gas is heated in the heat exchanger to a state corresponding to point 1' by absorbing heat from the forward stream (compressed hot gas). At the warm end of the heat exchanger, the difference in temperature between forward and return streams is $\Delta T_h = T_1 - T_{1'}$. The heat exchanger, expander, the condenser are all insulated from the surroundings.

The Nitrogen gas can be purified simply by passing it through a Pressure Swing Adsorption column system with Carbon molecular sieve (CMS) as an adsorbent. The purified Nitrogen is fed to the condenser through a pressure regulating valve at a pressure slightly above the atmospheric pressure so that gas can flow through its passage. The liquid nitrogen from the condenser passes to the liquid nitrogen storage vessel from which liquid nitrogen could be drawn through the delivery line as shown in (Figure 2). Evaporated nitrogen gas from the storage vessel is returned to the condenser for minimising losses and for optimizing the efficiency.

Equation for energy balances and for different important coefficients

First and second laws of thermodynamics are applied for finding energy balance and for practical case, total energy required (W) can be given by equation (1)

$$W = W_{\min} + T_0 \sum \Delta S \quad (1)$$

Where, $T_0 \sum \Delta S$ represent excess work in addition to minimum work (W_{\min}). Excess work is wasted as heat and is rejected to the surroundings. Entropy, being a quantitative characteristic of irreversibility, is used in evaluating energy losses and efficiency.

Cryogenic cycle analyses based on thermodynamic principle are reported in literature [30-38]. For the thermodynamic analysis of the cycle and for finding the energy balance equation the (Figure 2) could be reduced to a simple expander cycle as shown in (Figure 3a) and for the thermodynamic analysis corresponding T-S diagram is shown in (Figure 3b). The resulting thermodynamic cycle takes care of the necessary refrigeration load for liquefaction of nitrogen gas.

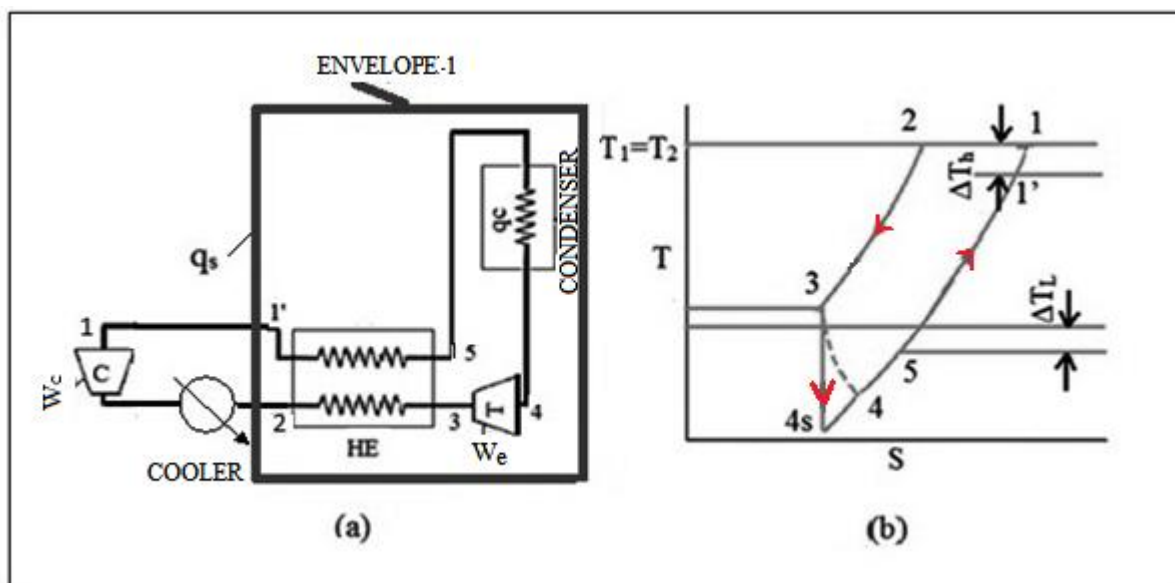


Figure 3 (a) Stirling cycle based Simple expander cycle, (b) T-S Diagram

Energy balance equations are generated around the envelope-1 as in (Figure 3a). For mass flow rate of M kg of compressed gas the energy balance equations can be written by neglecting variations in Kinetic and Potential energy of the system around the loop as:

$$Mh_2 + M(h_5 - h_4) + Mq_s = Mh_{1'} + MW_e \quad (2)$$

Where, h_1 , h_2 , h_3 , h_4 , h_5 are the specific enthalpies of the refrigerant gas at point 1', point 2, point 4 and point 5 respectively. h_s is the enthalpy difference in an ideal expander and is equal to isentropic enthalpy difference between points 3 and 4s in the T-S diagram (Figure 3b).

The total cooling capacity is given by

$$q_c = M(h_5 - h_4)$$

Now, substituting this term in equation (2) we get

$$Mh_2 + q_c + Mq_s = Mh_{1'} + Mh_s\eta_s \quad (3)$$

Again $h_{1'} = h_1 - C_p \Delta T_h$

Therefore, the equation (3) takes the form

$$Mh_2 + q_c + Mq_s = M(h_1 - C_p \Delta T_h) + Mh_s\eta_s \quad (4)$$

A Rearranging equation (4) we get,

$$\begin{aligned} Mh_s\eta_s + M(h_1 - h_2) &= q_c + MC_p \Delta T_h + Mq_s \\ \text{Or, } M(h_s\eta_s + h_1 - h_2) &= q_c + M(C_p \Delta T_h + q_s) \end{aligned} \quad (5)$$

For 1 kg of compressed gas (refrigerant) the equation (5) reduces to

$$\begin{aligned} h_s\eta_s + h_1 - h_2 &= q_c + (C_p \Delta T_h + q_s), \\ \text{which on rearranging becomes} \\ q_c &= (h_1 - h_2 + h_s\eta_s) - (q_s + C_p \Delta T_h) \end{aligned} \quad (6)$$

Ignoring the loss due to gas leaks, the specific work requirement of the cycle is

$$W = W_c - W_e\eta_m = \left(\frac{RT_1}{\eta_t}\right) \ln R_c - h_s\eta_s\eta_m \quad (7)$$

The compression ratio, $R_c = \frac{P_2}{P_1}$, is connected to the pressure ratio of the expander by a relation $R_c = jR_e$ and is given by

$$j = \frac{1 + \Delta P_3 / \Delta P_4}{1 - \Delta P_4 / P_4} \quad (8)$$

Where $R_e = \frac{P_3}{P_4}$, the temperature difference across the low temperature condenser is given by $\Delta T_L = T_L - T_5$. In theoretical cases, $T_L = 0$ i.e. T_L becomes equal to T_5 . For each value of T_L , there exist optimal cycle variables such that the coefficient of performance or the thermodynamic efficiency of the cycle η_t is maximum and optimisation of variables is necessary.

The various analysis techniques of exergetic losses can be performed for 1 kg of gas at different stages are as follows.

$$\Delta S_C = \frac{R}{\eta_T} \ln r_C - R \ln R_C = \left(\frac{1}{\eta_T} - 1\right) R \ln R_C \quad (9)$$

$$\Delta S_e = S_4 - S_3 + (h_3 \eta_s / T_0)(1 - \eta_m) \quad (10)$$

The last term on the right-hand side of equation (10) represents an additional increase in the entropy of the system due to loss of some of the work delivered by the expander, if any. For an ideal gas, equation (11) may be re-written.

$$\Delta S_e = C_p \left\{ \frac{\gamma - 1}{\gamma} \ln R_C + \frac{T_3 - T_4}{T_0} (1 - \eta_m) - \ln \frac{T_3}{T_4} \right\} \quad (11)$$

As for ideal gas we have,

$$h_s \eta_s = C_p (T_3 - T_4)$$

$$\eta_s = \frac{(T_3 - T_4)}{(T_3 - T_{4s})}$$

$$S_4 - S_3 = S_4 - S_{4s} = C_p \ln \frac{T_4}{T_{4s}}$$

$$\text{and, } \frac{T_4}{T_{4s}} = R_e = \frac{\gamma - 1}{\gamma(T_4/T_3)}$$

$$\Delta S_{HE} = C_p \left[\ln \left(1 - \frac{\Delta T_h}{T_1}\right) \left(1 + \frac{\Delta T_C}{T_5}\right) - \frac{\Delta T_C - \Delta T_h}{T_1} \right] \quad (12)$$

Where $\Delta T_C = T_3 - T_5$ and $\Delta T_h = T_1 - T_1'$,

$$\Delta S_{pd} = \frac{\gamma - 1}{\gamma} C_p \ln j \quad (13)$$

Total increase in entropy in the heat exchanger stage is

$$\Delta S_H = \Delta S_{HE} + \Delta S_{pd} \quad (14)$$

$$\Delta S_L = C_p \left(\ln \frac{T_5}{T_4} - \frac{T_5 - T_4}{T_L} \right) \quad (15)$$

$$\Delta S_{add} = \frac{1}{2} C_p \left(\frac{\Delta T_h}{T_1} \right)^2 \quad (16)$$

If the entropy changes are known, it is possible to determine the exergetic losses,

$$L_i = T_0 \Delta S_i \quad (17)$$

or the reduced exergetic losses,

$$T_0 \frac{\Delta S_i}{W} = \overline{L}_i \quad (18)$$

in the various steps of the cycle.

The reduced exergetic losses of the various steps of the cycles are given by

$$\frac{T_0 \Delta S_C}{W} = \overline{L}_C \quad (19)$$

$$\frac{T_0 \Delta S_e}{W} = \bar{L}_e \quad (20)$$

$$\frac{T_0 \Delta S_H}{W} = \frac{T_0 (\Delta S_{HE} + \Delta S_{pd})}{W} = \bar{L}_H \quad (21)$$

$$\frac{T_0 \Delta S_L}{W} = \bar{L}_L \quad (22)$$

$$\frac{T_0 \Delta S_{add}}{W} = \bar{L}_{add} \quad (23)$$

Therefore, total reduced exergetic losses in all steps making up the cycle is given by

$$\sum \bar{L}_i = \bar{L}_C + \bar{L}_e + \bar{L}_H + \bar{L}_L + \bar{L}_{add} \quad (24)$$

And, thermodynamic efficiency of the cycle is given by

$$\eta_t = 1 - \sum \bar{L}_i \quad (25)$$

Cycle calculation

With reference to Figure 2 and based on the following assumption, cycle calculation and exergetic analysis could be carried out to find out the overall thermodynamic efficiency, coefficient of performance of the cycle, specific work requirement and specific cooling capacity etc.

Assumptions are as follows:

- $\Delta T_h = 4\text{K}$ and $\Delta T_c = 5\text{K} = (T_3 - T_5)$
- $\eta_s = 0.8$
- $\eta_t = 0.75$
- $j = 1.1$
- $\eta_m = 0$, Work of compression is not returned to the cycle
- Nitrogen is assumed to enter the condenser at 1.2 atm. (0.12MPa)
- Ambient temperature $T_0 = T_1 = T_2 = 300\text{K}$

The above assumptions and conditions are used in subsequent development and cycle calculation for Hydrogen and Helium expander cycle for Nitrogen liquefaction system. Expander ratio is taken as 20 so that the refrigerant gas operates “between 1 to 22” (0.1 MPa to 2.2MPa). Temperature drop of 60K (assumed) due to expansion for practical purposes though higher temperature drop is calculated from T-S diagram.

Cycle Calculation based on expander refrigeration cycle using Helium gas as the refrigerant

Temperatures at various points are as follows

$$\begin{aligned} T_0 &= T_1 = T_2 = 300\text{K} \\ T_3 &= 135\text{K}, T_4 = 75\text{K}, T_5 = T_1 = 130\text{K} \\ \text{Gas constant } R &= 2078.5\text{ J/kg/K}, \\ C_p &= 5.2 \times 10^3\text{ J/kg/K} = 5.2\text{ KJ/kg/K} \\ \gamma &= 1.66. \end{aligned}$$

Calculations

The specific work requirement

$$W = \left(\frac{RT_1}{\eta_T} \right) \ln r_c - h_s \eta_s \eta_m = \left(\frac{RT_1}{\eta_T} \right) \ln r_c \quad (\eta_m = 0)$$

$$= \frac{2078.5 \times 300}{0.75} \ln(22) = 2569892.7 \text{ J/kg} = 2569.8927 \text{ kJ/kg of refrigerant gas}$$

Heat in leak into the system, $q_s = 6 \text{ kJ/kg} = 6000 \text{ J/kg}$ (assumed).

Specific cooling capacity, $q_c = (h_1 - h_2 + h_s \eta_s) - (q_s + C_p \Delta T_h)$

From T-S diagram of helium

$$h_1 (T_1 = 300 \text{ K}, P_1 = 1 \text{ atm}) = 1600000 \text{ J/kg/K} = 1600 \text{ kJ/kg/K}$$

$$h_2 (T_2 = 300 \text{ K}, P_2 = 22 \text{ atm}) = 1500000 \text{ J/kg/K} = 1500 \text{ kJ/kg/K}$$

$$h_s (\text{ideal isentropic drop}) = 652500 \text{ J/kg/K} = 652.5 \text{ kJ/kg/K}$$

Specific cooling capacity

$$= (1600 - 1500 + 652.5 \times 0.80) - (6 + 5.2 \times 4) \\ = 497.2 \text{ kJ/kg of refrigerant gas}$$

Coefficient of performance, (C.O.P)

$$\frac{q_c}{W} = \frac{497.2}{2569.89} = 0.1935 \text{ i.e. } 19.35\%$$

For a 10 kg/hr capacity nitrogen liquefaction plant amount of refrigerant required per hour = 3250 kJ (calculated from T-S diagram) and considering condenser efficiency of 80%, amount of refrigerant to be used = $3250 / 0.80 = 4062.5 \text{ kJ}$

Amount of refrigerant gas (Helium) required = $4062.5 / 497.2 = 8.17 \text{ kg}$

Work requirement for the purpose = $8.17 \times 2569.89 = 20997.94 \text{ kJ/hr}$

Work requirement per kg of liquid nitrogen product = $20997.94 / 10 = 2099.794 \text{ kJ}$

Cycle Calculation based on expander refrigeration cycle using Hydrogen gas as the refrigerant

Temperatures at various points are as follows

$$T_0 = T_1 = T_2 = 300 \text{ K}$$

$$T_3 = 135 \text{ K}, T_4 = 75 \text{ K}, T_5 = T_L = 130 \text{ K}$$

Gas constant $R = 4157 \text{ J/kg/K}$,

$C_p = 10510 \text{ J/kg/K} = 10.51 \text{ kJ/kg/K}$,

$\gamma = 1.41$.

Calculations:

The specific work requirement

$$W = \left(\frac{RT_1}{\eta_T} \right) \ln r_c - h_s \eta_s \eta_m = \left(\frac{RT_1}{\eta_T} \right) \ln r_c \quad (\eta_m = 0) \\ = \frac{4157 \times 300}{0.75} \ln(22) = 5139785.39 \text{ J/kg} = 5139.78 \text{ kJ/kg of refrigerant gas}$$

Heat in leak into the system, $q_s = 6 \text{ kJ/Kg} = 6000 \text{ J/kg}$ (assumed).

Specific cooling capacity, $q_c = (h_1 - h_2 + h_s \eta_s) - (q_s + C_p \Delta T_h)$

From T-S diagram of hydrogen

$$h_1 (T_1 = 300 \text{ K}, P_1 = 1 \text{ atm}) = 4284000 \text{ J/kg/K} = 4284 \text{ kJ/kg/K}$$

$$h_2 (T_2 = 300 \text{ K}, P_2 = 22 \text{ atm}) = 4200000 \text{ J/kg/K} = 4200 \text{ kJ/kg/K}$$

$$h_s (\text{ideal isentropic drop}) = 1281000 \text{ J/kg/K} = 1281 \text{ kJ/kg/K}$$

Specific cooling capacity

$$= (4284 - 4200 + 1281 \times 0.80) - (6 + 10.51 \times 4)$$

= 1060 kJ/kg of refrigerant gas

Coefficient of performance, (C.O.P)

$$\frac{q_c}{W} = \frac{1060}{5139.78} = 0.206 \text{ i.e. } 20.6\%$$

For a 10 kg/hr capacity nitrogen liquefaction plant amount of refrigerant required per hour = 3250 kJ (calculated from T-S diagram) and considering condenser efficiency of 80%, amount of refrigerant to be used = 4062.5 kJ

Amount of refrigerant gas (Hydrogen) required

$$= 4062.5 / 1060 = 3.832 \text{ kg}$$

Work requirement for the purpose

$$= 3.832 \times 5139.78 = 19695.63 \text{ kJ/hr}$$

Work requirement per kg of liquid nitrogen product

$$= 19695.63 / 10 = 1969.56 \text{ kJ}$$

Increase in Entropy calculation for per Kilogram of Refrigerant used

Entropy increase per kg of refrigerant in different stages as computed is given below.

Helium refrigerant

Compression stage

$$\Delta S_c = \frac{R}{\eta_r} \ln r_c - R \ln r_c = \left(\frac{1}{\eta_r} - 1\right) R \ln r_c = \left(\frac{1}{0.75} - 1\right) \times 2078.5 \times \ln 22$$

$$= 2120.16 \text{ J / kg / K}$$

Expansion stage

$$\Delta S_e = C_p \left\{ \frac{\gamma - 1}{\gamma} \ln r_e + \frac{T_3 - T_4}{T_0} (1 - \eta_m) - \ln \frac{T_3}{T_4} \right\}$$

$$= 5.2 \times 10^3 \left\{ \frac{1.66 - 1}{1.66} \ln 20 + \frac{135 - 75}{300} (1 - 0) - \ln \frac{135}{75} \right\}$$

$$= 4165.20 \text{ J / kg / K}$$

Heat exchanger stage

$\Delta S_H = \Delta S_{HE} + \Delta S_{pd}$ where ΔS_{HE} is the increase in entropy due to heat transfer and in ΔS_{pd} is the increase in entropy due to pressure drop.

$$\Delta S_{HE} = C_p \left[\ln \left(1 - \frac{\Delta T_h}{T_1}\right) \left(1 + \frac{\Delta T_c}{T_5}\right) - \frac{\Delta T_c - \Delta T_h}{T_1} \right]$$

$$= 5.2 \times 10^3 \left[\ln \left(1 - \frac{4}{300}\right) \left(1 + \frac{5}{130}\right) - \frac{5 - 4}{300} \right] = 110.58 \text{ J / kg / K}$$

$$\Delta S_{pd} = \frac{\gamma - 1}{\gamma} C_p \ln j = \frac{1.66 - 1}{1.66} \times 5.2 \times 10^3 \times \ln 1.15 = 197.05 \text{ J / kg / K}$$

$$\Delta S_H = \Delta S_{HE} + \Delta S_{pd}$$

$$= 110.58 + 197.05 = 307.63 \text{ J / kg / K}$$

Condenser stage

$$\Delta S_L = C_p \left(\ln \frac{T_5}{T_4} - \frac{T_5 - T_4}{T_L} \right) = 5.2 \times 10^3 \left(\ln \frac{130}{75} - \frac{130 - 75}{130} \right) = 655.2 J / kg / K$$

Irreversibility of the process (1-1'):

$$\Delta S_{add} = \frac{1}{2} C_p \left(\frac{\Delta T_h}{T_1} \right)^2 = \frac{1}{2} \times 5.2 \times 10^3 \left(\frac{4}{300} \right)^2 = 0.46 J / kg / K$$

*Hydrogen refrigerant**Compression stage*

$$\begin{aligned} \Delta S_C &= \frac{R}{\eta_T} \ln r_C - R \ln r_C = \left(\frac{1}{\eta_T} - 1 \right) R \ln r_C = \left(\frac{1}{0.6} - 1 \right) \times 4157 \times \ln 22 \\ &= 4277.42 J / kg / K \end{aligned}$$

Expansion stage

$$\begin{aligned} \Delta S_e &= C_p \left\{ \frac{\gamma - 1}{\gamma} \ln r_e + \frac{T_3 - T_4}{T_0} (1 - \eta_m) - \ln \frac{T_3}{T_4} \right\} \\ &= 10510 \left\{ \frac{1.41 - 1}{1.41} \ln 20 + \frac{135 - 75}{300} (1 - 0) - \ln \frac{135}{75} \right\} \\ &= 5044.8 J / kg / K \end{aligned}$$

Heat exchanger stage

$\Delta S_H = \Delta S_{HE} + \Delta S_{pd}$ where ΔS_{HE} is the increase in entropy due to heat transfer and in ΔS_{pd} is the increase in entropy due to pressure drop.

$$\begin{aligned} \Delta S_{HE} &= C_p \left[\ln \left(1 - \frac{\Delta T_h}{T_1} \right) \left(1 + \frac{\Delta T_C}{T_5} \right) - \frac{\Delta T_C - \Delta T_h}{T_1} \right] \\ &= 10510 \left[\ln \left(1 - \frac{4}{300} \right) \left(1 + \frac{5}{130} \right) - \frac{5 - 4}{300} \right] = 214.26 J / kg / K \end{aligned}$$

$$\Delta S_{pd} = \frac{\gamma - 1}{\gamma} C_p \ln j = \frac{1.41 - 1}{1.41} \times 10510 \times \ln 1.1 = 290.49 J / kg / K$$

$$\begin{aligned} \Delta S_H &= \Delta S_{HE} + \Delta S_{pd} \\ &= 214.26 + 290.49 = 504.75 J / kg / K \end{aligned}$$

Condenser stage

$$\Delta S_L = C_p \left(\ln \frac{T_5}{T_4} - \frac{T_5 - T_4}{T_L} \right) = 10510 \left(\ln \frac{130}{75} - \frac{130 - 75}{130} \right) = 1366.31 J / kg / K$$

Irreversibility of the process (1-1')

$$\Delta S_{add} = \frac{1}{2} C_p \left(\frac{\Delta T_h}{T_1} \right)^2 = \frac{1}{2} \times 10510 \left(\frac{4}{300} \right)^2 = 0.93 J / kg / K$$

Evaluation of Overall Thermodynamic Efficiency based on Exergetic Losses
Helium refrigerant

$$\frac{T_0 \Delta S_C}{W} = 0.247 = \overline{L}_C$$

$$\frac{T_0 \Delta S_e}{W} = 0.486 = \overline{L}_e$$

$$\frac{T_0 \Delta S_H}{W} = 0.036 = \overline{L}_H$$

$$\frac{T_0 \Delta S_L}{W} = 0.076 = \overline{L}_L$$

$$\frac{T_0 \Delta S_{add}}{W} = 0.000054 = \overline{L}_{add}$$

Therefore, $\sum \overline{L}_i = (\overline{L}_C + \overline{L}_e + \overline{L}_H + \overline{L}_L + \overline{L}_{add}) = 0.844$

Then, overall thermodynamic efficiency of the cycle is given by,

$$\eta_T = 1 - \sum \overline{L}_i = 0.155 \text{ i.e. } 15.5\%$$

Hydrogen refrigerant

$$\frac{T_0 \Delta S_C}{W} = 0.249 = \overline{L}_C$$

$$\frac{T_0 \Delta S_e}{W} = 0.294 = \overline{L}_e$$

$$\frac{T_0 \Delta S_H}{W} = 0.029 = \overline{L}_H$$

$$\frac{T_0 \Delta S_L}{W} = 0.079 = \overline{L}_L$$

$$\frac{T_0 \Delta S_{add}}{W} = 0.000075 = \overline{L}_{add}$$

Therefore, $\sum \overline{L}_i = (\overline{L}_C + \overline{L}_e + \overline{L}_H + \overline{L}_L + \overline{L}_{add}) = 0.651$

Then, overall thermodynamic efficiency of the cycle is given by,

$$\eta_T = 1 - \sum \overline{L}_i = 0.366 \text{ i.e. } 36.6\%$$

Result and Discussion

From **Table 1**, it can be observed that overall thermodynamic efficiency is almost double for Hydrogen expander cycle as compared to Helium expander cycle under the same condition operation although in practice a higher temperature drop is expected for Helium expander cycle for the same adiabatic efficiency of the expander. On the other hand the coefficient of performance (C.O.P) of helium expander cycle is slightly lower in comparison to that of Hydrogen expander cycle. The work requirement per kg of liquid Nitrogen product (W_L) comes to 2099.79 kJ in case of helium expander cycle as compared to hydrogen expander cycle ($W_L = 1969.56$ kJ). Moreover hydrogen gas shows more ideal behaviour than helium gas. On the basis of cycle calculation it is observed that specific cooling capacity of Helium is 497.20 kJ/kg which is quite less than Hydrogen (1060kJ/kg).

From **Table 2**, it can be observed that at every stage, per kg of Hydrogen refrigerant used shows higher entropy value compared to per kg of Helium refrigerant used for nitrogen liquefaction.

Table 3, shows reduced exergetic losses for Hydrogen expander and Helium Expander Cycle. From the table it can be concluded that exergetic losses are less for Hydrogen refrigerant compared to Helium refrigerant. It also seen that most losses occurs in compression and expander stages for both the refrigerant gases.

Table 1 Result of Thermodynamic Analysis

Type of Cycle	Hydrogen expander cycle	Helium expander Cycle
Specific Cooling capacity	1060kJ/kg	497.20 kJ/kg
Coefficient of performance	20.60 %	19.35 %
Work requirement per kg of liquid nitrogen product (W_L)	1969.56 kJ	2099.79 kJ
Overall thermodynamic efficiency	36.6 %	15.5 %

Table 2 Result of various entropy increases per kg of refrigerants used

Type of Cycle	Hydrogen expander cycle (J/kg/K)	Helium expander Cycle (J/kg/K)
ΔS_C	4277.42	2120.16
ΔS_e	5044.81	4165.20
$\Delta S_H = \Delta S_{HE} + \Delta S_{pd}$	504.75	307.63
(ΔS_L)	1366.33	655.20
(ΔS_{add})	0.934	0.461

Table 3 Result of reduced exergetic losses of two refrigerant cycles

Type of Cycle	Hydrogen expander cycle	Helium expander Cycle
$\overline{L_C}$	0.249	0.247
$\overline{L_e}$	0.294	0.486
$\overline{L_H}$	0.029	0.036
$\overline{L_L}$	0.079	0.076
$\overline{L_{add}}$	0.000075	0.000054

Conclusion

Thus from the foregoing discussion it can be concluded that due to several advantage of hydrogen cycle, the high pressure hydrogen expander refrigeration cycle would be the best choice for development of Nitrogen liquefaction cycle for small to medium scale nitrogen liquefaction plant based on Reverse Stirling Refrigeration Cycle. It appears from the exergetic analysis that the improvement in efficiencies of compression, expander stages is of utmost importance as exergetic losses in these stages are more. Proper insulation and maintenance can be carried out at every stages of expander refrigerant cycle especially in the compression and expander along with heat exchanger and all other stages to minimise further exergetic losses. The critical thermodynamic analysis undertaken in this article will help as a guide line for energy savings measures to be taken in small scale cryogenic plant.

Nomenclature

W_{min} Min work required in case of ideal cycle

T_0 Temperature of the surroundings

$\sum \Delta S$ The total entropy changes of all bodies

q_s The heat load due to imperfect insulation

W_e The expander work and is equal to $h_s \eta_s$

η_s Adiabatic efficiency of the expander

C_p Specific heat of gas

ΔT_h Temperature difference at the warm end of the heat exchanger

ΔT_c Temperature difference at the Cold end of the heat exchanger

q_c The specific cooling capacity

R_c Compression ratio of the compressor

R_e Expansion ratio

η_i Isothermal efficiency of the compressor

η_m Mechanical efficiency of the expander

R The gas constant

j The reduced pressure drop coefficient

$\Delta P_3 / \Delta P_4$ Dimensionless pressure drops for the forward

$\Delta P_4 / P_4$ The dimensionless pressure drops for the return stream.

T_L The constant temperature in the low temperature condenser

S_3 and S_4 The specific entropy of the working fluid (refrigerant)

γ The specific heat ratio of the refrigerant gas

W Specific work requirement of the cycle

ΔS_c Increase in entropy of the system in the compressor stage

ΔS_e Increase in entropy of the system in the adiabatic turbo expander stage

ΔS_H Increase in the entropy of the system in the heat exchanger stage

ΔS_L Increase in the entropy of the system in the low temperature chamber stage

ΔS_{add} Increase in the entropy of the system to a small additional loss due to irreversibility of the process (1-1')stage

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